THE QUOTIENT-REMAINDER THEOREM AND THE DEFINITIONS OF (n mod d) and (n div d)

THEOREM 4.4.1: THE QUOTIENT-REMAINDER (Q-R) THEOREM

FOR every integer n and every positive integer d, there exist UNIQUE integers q and r such that

$$n = dq + r \quad \text{and} \quad 0 \leq r < d.$$

DEFINITIONS OF (n mod d) and (n div d)

Let d be a positive integer and let n be any integer. By the Q-R Theorem, there exist unique integers q and r such that $n = dq + r$ and $0 \leq r < d$.

Then $(n \bmod d)$ is defined to be $(n \bmod d) = r$

and
$(n \operatorname{div} d)$ is defined to be $(n \operatorname{div} d) = q$.

Thus, since $23 = 4 \times 5 + 3$ and $0 \leq 3 < 4$,
$(23 \bmod 4) = 3$ and $(23 \operatorname{div} 4) = 5$.

Since $-29 = 7 \times (-5) + 6$ and $0 \leq 6 < 7$,
$(-29 \bmod 7) = 6$ and $(-29 \operatorname{div} 7) = -5$.

In this class, always write "$(n \bmod d)$" using parentheses!
Don't write "$16 \bmod 5 = 1$". Instead write "$(16 \bmod 5) = 1$".